



Circles

Circle

A circle is the locus of a points which moves in a plane in such a way that its distance from a fixed point remains constant.

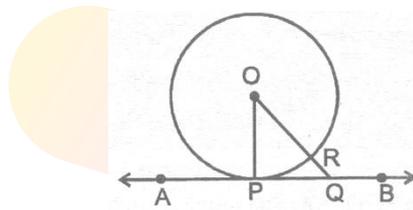
Secant and Tangent :

⇒ Secant to a circle is a line which intersects the circle in two distinct points.

⇒ A tangent to a circle is a line that intersects the circle in exactly one point.

Theorem:

Statement : A tangent to a circle is perpendicular to the radius through the point of contact.



Given : A circle C (O, r) and a tangent AB at a point P.

To prove : $OP \perp AB$

Construction : Take any points Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R.

Proof: Among all line segments joining the point O to a point on AB, the shortest one is perpendicular to

AB. So, to prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment

joining O to any point of AB.

Clearly $OP = OR$

Now, $OQ = OR + RQ$

⇒ $OQ > OR$

⇒ $OQ > OP (\because OP = OR)$

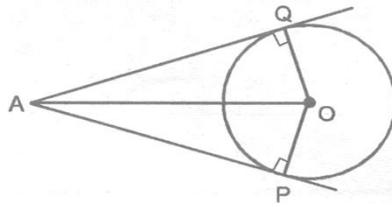
Thus, OP is shorter than any other segment joining O to any point of AB.

Hence, $OP \perp AB$.

Theorem:



Statement : Lengths of two tangents drawn from an external point to a circle are equal.



Given: AP and AQ are two tangents drawn from a point A to a circle C (O, r).

To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : In ΔAOQ and ΔAPO

$\angle OQA = \angle OPA$ [Tangent at any point of a circle is perp. to radius through the point of contact]

AO = AO [Common]

OQ = OP [Radius]

So, by R.H.S. criterion of congruency $\Delta AOQ \cong \Delta AOP$

\therefore AQ = AP [By CPCT] Hence Proved.

Result:

- (i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. $\angle OAQ = \angle OAP$ [By CPCT]
- (ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle OAQ = \angle OAP$ [By CPCT]

Example: If all the sides of a parallelogram touches a circle, show that the parallelogram is a rhombus.

Sol. Given : Sides AB, BC, CD and DA of a \parallel^{gm} ABCD touch a circle at P, Q, R and S respectively.

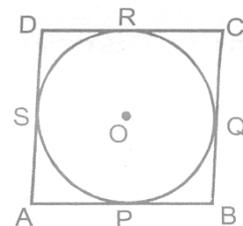
To prove \parallel^{gm} ABCD is a rhombus.

Proof : AP = AS(i)

BP = BQ(ii)

CR = CQ(iii)

DR = DS(iv)



[Tangents drawn from an external point to a circle are equal]

Adding (1), (2), (3) and (4), we get



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$$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

$$\Rightarrow AB + AB = AD + AD \quad [\text{In a } \parallel^{\text{gm}} \text{ ABCD, opposite side are equal}]$$

$$\Rightarrow 2AB = 2AD \text{ or } AB = AD$$

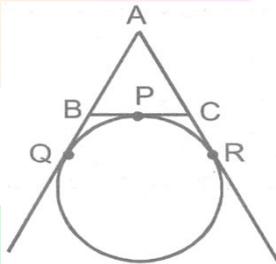
But $AB = CD$ AND $AD = BC$ [Opposite sides of a \parallel^{gm}]

$$\therefore AB = BC = CD = DA$$

Hence, \parallel^{gm} ABCD is a rhombus.

Example: A circle touches the BC of a ΔABC at P and touches AB and AC when produced at Q and R respectively as shown in figure, Show that $AQ = \frac{1}{2}$ (Perimeter of ΔABC).

So. Given : A circle is touching side BC of ΔABC at P and touching AB and AC when produced at Q and R respectively.



To prove : $AQ = \frac{1}{2}$ (perimeter of ΔABC)

Proof : $AQ = AR$ (i)

$BQ = BP$ (ii)

$CP = CR$ (iii)

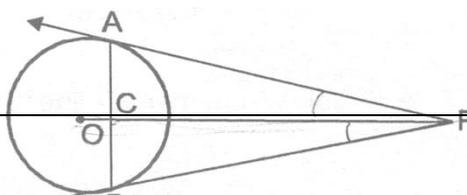
[Tangents drawn from and external point to a circle are equal]

$$\begin{aligned} \text{Now, perimeter of } \Delta ABC &= AB + BC + CA \\ &= AB + BP + PC + CA \\ &= (AB + BQ) + (CR + CA) \quad [\text{From (ii) and (iii)}] \\ &= AQ + AR = AQ + AQ \quad [\text{From (i)}] \end{aligned}$$

$$AQ = \frac{1}{2} \text{ (perimeter of } \Delta ABC).$$

Examples: Prove that the tangents at the extremities of any chord make equal angles with the chord.

Sol. Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose, the tangents meet at point P. Join OP. Suppose OP meets AB at C.





We have to prove that

$$\angle PAC = \angle PBC$$

In triangles PCA and PCB

$$PA = PB \quad [\because \text{Tangent from an external point are equal}]$$

$$\angle APC = \angle BPC \quad [\because \text{PA and PB are equally inclined to OP}]$$

$$\text{And } PC = PC \quad [\text{Common}]$$

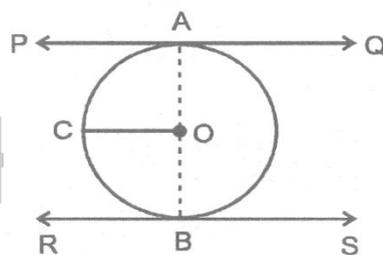
So, by SAS criteria of congruence

$$\triangle PAC \cong \triangle PBC$$

$$\Rightarrow \angle PAC = \angle PBC \quad [\text{By CPCT}]$$

Examples: Prove that the segment joining the points of contact of two parallel tangents passes through the centre.

Sol. Let PAQ and RBS be two parallel tangents to a circle with centre O. Join OA and OB. Draw OC || PQ. Now, PA || CO



$$\Rightarrow \angle PAO + \angle COA = 180^\circ \quad [\text{Sum of co-interior angle is } 180^\circ]$$

$$\Rightarrow 90^\circ + \angle COA = 180^\circ \quad [\because \angle PAO = 90^\circ]$$

$$\Rightarrow \angle COA = 90^\circ$$

Similarly, $\angle COB = 90^\circ$

$$\therefore \angle COA + \angle COB = 90^\circ + 90^\circ = 180^\circ$$

Hence, AOB is a straight line passing through O.